

MTH 111, Math for the Architects , Exam One

Ayman Badawi

QUESTION 1. (12 points) Find the focus, the vertex, and the directrix for the parabola $8y = 4x^2 + 8x + 20$. Let F be the focus of the given parabola. Given $A = (1, 4)$ lies on the parabola. Find $|AF|$. Drew a rough graph of the given parabola.

QUESTION 2. (12 points) Given $(3, 9)$ and $(3, -7)$ are the foci of a hyperbola and $K = 6$ is its constant. Write down the standard form equation of the hyperbola. Sketch a rough graph of the hyperbola.

QUESTION 3. (12 points) Find the foci, the center, and the constant K for the ellipse $9x^2 + 5y^2 + 20y - 25 = 0$. Sketch a rough graph of the ellipse.

QUESTION 4. (6 points) Does the line $y = x + 2$ intersect the hyperbola $y^2 - (x - 1)^2 = 3$? If yes, find the intersection points.

QUESTION 5. a) **(10 points)** Find the equation of the line that is perpendicular to the line $3y+4x=2$ and it passes through the point $(4, 1)$.

b)(**5 points)** Given $L : 2x + 3y = 13$ and $A = (4, 6)$ is a point not on the line L. Find the distance between A and L.

~~**QUESTION 6.** (i) **(5 points)**~~ $\cancel{Lim_{x\to\ -1}\ \frac{\sqrt{x+10}-3}{x^2-1}}$

~~(ii) **(5 points)**~~ $\cancel{Lim_{x\to\ -2^+}\ \frac{x+3}{x^2-4}}$

~~**Faculty information**~~

~~Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.~~
~~E-mail: abadawi@aus.edu, www.ayman-badawi.com~~

1. (5pts) Find the equation of the line that passes through the point $(1, 2)$ and is perpendicular to the line $6x - 2y = 3$.

2. (5pts) Find the value of k such that the lines

$$\mathbf{r}_1(t) = \langle 4, -9, 1\rangle + t\langle 2, k, 1\rangle$$
$$\mathbf{r}_2(t) = \langle 4, -9, 1\rangle + t\langle 4, 6, 2\rangle$$

are parallel

3. (5pts) Find the parametric equation of the line passing through the points $(1, 0, 4)$ and $(1, 1, 2)$.

4. (5pts) Find the equation of the plane containing the points $(0, 0, 1)$, $(2, 0, 1)$ and $(1, 1, 2)$

5. (5pts) Find the equation of the line which passes through the point $(3, 5, 7)$ and is perpendicular to the plane $2x + 4y + 6z = 8$

6. (10pts) For each of the following equations state the shape of the curve it determines (eg, line, circle, etc). You do NOT have to provide any reasoning.

   (a) $x + y = 4y$

   (b) $4x + y^2 = 7$

   (c) $y^2 = 4 + x^2$

   ~~(d) Polar equation $r = \theta$.~~

   ~~(e) Polar equation $r = 2 + \cos(\theta)$.~~

7. (5pts) Find the equation of the hyperbola so that: it is centered at the origin, one of the focal points is $(0, 5)$, and the distance between the axis intercepts is 6.

~~8. (5pts) Carefully sketch the polar curve~~ $\cancel{r = \cos(\theta) + \sin(\theta)}$~~.~~

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MTH 111, Review Math for Architects

Ayman Badawi

QUESTION 1. Let $u = -2i + 3j - 4k$, $v = i + k$, $w = 2i - j + 5k$.

a) How many planes are there where each contains u and v and the point (1, 3, 2)? Find them all.

b) Is there a plane containing u, v, w? if not then find the volume of the twisted cube formed by u, v, w.

c) Find $Proj_w^u$ and then $|Proj_w^u|$.

d) Find the area of the triangle that has vertices : $(2, 5), (0, 8), (-3, 1)$

e) Find the area of the triangle that has vertices (1, 1, 2), (2, -1, 1), and (0, 0, 4)

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Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

MTH 111, Math for Architects,EXAM II REVIEW, Spring 2013

Ayman Badawi

QUESTION 1. Given the points: $A = (2, 8)$ and $B = (0, 6)$. Find a point C on the line y = 3 so that $|AC| + |CB|$ is minimum. You need to find the coordinates of the point C.

QUESTION 2. Find y' and don't simplify :

a) $y = 3e^{(2x+4)} + ln(7x^2 + 8x + 7) + 10xe^{2x+1}$

b) $y = 4x(7x^3 + 2e^x)^3 + \sqrt{4x + 9} + \frac{7}{x^3}$

c) $y = \sqrt[3]{7x + 1} + \frac{e^{(3x+1)}}{ln(5x+2)}$

d) $y = \frac{7x+2}{x^2-12x+3}$

e) $y = ln[(3x + 2)^3(7x^2 + 8x - 9)^8]$

f) $y = ln[\frac{8x^2+7x-9}{4x+e^{2x}}] + 3x^2 - 45x$

QUESTION 3. a) Find $lim_{x\to 2} \ \frac{e^{(2x-4)}-1}{3x^2-3x-6}$

b) Find $Lim_{x\to -3} \ \frac{\sqrt[3]{3x+1}+2}{7x+21}$

c) $Lim_{x\to -3} \ \frac{\sqrt[3]{3x+1}+4}{7x+23}$

d) $Lim_{x\to -3} \ \frac{ln(3x+10)}{e^{3x+9}-2x-7}$

QUESTION 4. Let $f(x) = 4xe^{2x-3} + 3\sqrt{8x-3} + ln(6x-8) - 1$

a) Find the equation of the tangent line to the curve of $f(x)$ when $x = 1.5$.

b) Find the actual value for $f(1.8)$ [you may want to use a calculator]

c) Use (a) to approximate $f(1.8)$.

QUESTION 5. a)Given $e^{2x-10} + ln(2x+3y) = -yx - 14$. Find the equation of the tangent line to the curve at $(5, -3)$.

b) Approximate the y-value when $x = 5.2$

QUESTION 6. a) Find all local min and local max of y where $y = -x^2e^x + 3e^x + 1$.

b) For what values of x does y increase? for what values of x does y decrease?

c) Let y as above but defined on [-4, 2] (i.e., $-4 \leq x \leq 2$). Find the absolute Max value of y and the absolute min of y.

QUESTION 7. Find two numbers A, B where $A + 2B = 15$ and AB is maximum.

QUESTION 8. We want to construct a rectangle with maximum area such that two vertices on the line y = 12 and the other two vertices on the curve $y = x^2$. What should be the length and the width of such rectangle?

QUESTION 9. Evaluate the following integrals:

a) $\int 7xe^{x^2+1} + \sqrt{x} + 4x \, dx$

a/2) $\int \frac{2x+1}{x^2+x+3} \, dx$

a/3) $\int \frac{x^3+x^2-6}{x^7} \, dx$

a/4) $\int (e^x + 1)\sqrt{e^x + x + 4} \, dx$

a/5) $\int \frac{1}{e^{-x}+8}$

a/6) $\int \frac{2xe^{x^2}}{(e^{x^2}+4)^4} \, dx$

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MTH 111, Math for Architects, Exam I Spring 2013

Ayman Badawi

QUESTION 1. a) Find an equation of the plane that contains the point $(1, 2, 0)$ and the two vectors : $2i+j-k,\ -3i+2j+5k$.

b) Can we draw the line: $x = 1 + 6t, y = 2 + 3t, z = -3k$ inside the plane in (a)? EXPLAIN CLEARLY.

c) Given a parametric equations of two lines L_1 and L_2, where $L_1 : x = 1 + t, y = 4 - 2t, z = 2 + t$ and $L_2 : x = -2s, y = 9 + s, z = 5 - 6s$. Is L_1 perpendicular to L_2? If the two lines intersect, then find the intersection point.

QUESTION 2. a) Given the plane $P : 3x + y - z = 14$ and $Q = (1, 7, 7)$ not on the plane P. Find the distance between Q and P.

b) Given the line $L_1 : x = 3+2t, y = 4+t, z = 2-3t$ and $Q = (5, -3, 7)$ not on the line L_1. Find the distance between Q and L_1.

c) Given $V = i - 2j - 2k$. Find two vectors W, F such that W and F are parallel to V , $W \neq F$ but $|W| = |F| = 8.6$

d) Given $V = 6i - 6j + 3k$ and $W = 2i - j + 2k$. Find $Proj_V^W$ and $|Proj_V^W|$. If θ is the angle between V and W what is $cos(\theta)$?

QUESTION 3. a)Find the area of the triangle that has vertices: $(1, 1, 1), (1, 2, 5), (2, 2, 7)$

b)Find the vertex, the directrix and the focus of $8x = y^2 - 8y + 48$. Give a rough sketch of the parabola.

c) Find the center, the foci and the constant k of the ellipse $4x^2 + 8x + y^2 + 2y + 1 = 0$. Give a rough sketch of the ellipse.

QUESTION 4. a) Find the center, the foci, the constant k of the hyperbola $x^2 - 4x - 9y^2 - 18y - 14 = 0$. Give a rough sketch of the hyperbola.

b) Find the equation of the hyperbola that has $(4, 6), (4, -2)$ as its foci, and one of its vertices is $(4, 4)$. Give a rough sketch of the hyperbola.

QUESTION 5. Given the points: $A = (2, 8)$ and $B = (0, 6)$. Find a point C on the line y = 3 so that $|AC| + |CB|$ is minimum. You need to find the coordinates of the point C.

QUESTION 6. Find y' and don't simplify :

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b) $y = 4x(7x^3 + 2e^x)^3 + \sqrt{4x+9} + \frac{7}{x^3}$

c) $y = \sqrt[3]{7x+1} + \frac{e^{(3x+1)}}{ln(5x+2)}$

d) $y = \frac{7x+2}{x^2-12x+3}$

e) $y = ln[(3x+2)^3(7x^2+8x-9)^8]$

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b) Find $Lim_{x\to -3} \ \frac{\sqrt[3]{3x+1}+2}{7x+21}$

c) $Lim_{x\to -3} \ \frac{\sqrt[3]{3x+1}+4}{7x+23}$

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a) Find the equation of the tangent line to the curve of $f(x)$ when $x = 1.5$.

b) Find the actual value for $f(1.8)$ [you may want to use a calculator]

c) Use (a) to approximate $f(1.8)$.

QUESTION 9. a)Given $e^{2x-10} + ln(2x+3y) = -yx - 14$. Find the equation of the tangent line to the curve at $(5, -3)$.

b) Approximate the y-value when $x = 5.2$

QUESTION 10. a) Find all local min and local max of y where $y = -x^2e^x + 3e^x + 1$.

b) For what values of x does y increase? for what values of x does y decrease?

c) Let y as above but defined on [-4, 2] (i.e., $-4 \le x \le 2$). Find the absolute Max value of y and the absolute min of y.

QUESTION 11. Find two numbers A, B where $A + 2B = 15$ and AB is maximum.

QUESTION 12. We want to construct a rectangle with maximum area such that two vertices on the line y = 12 and the other two vertices on the curve $y = x^2$. What should be the length and the width of such rectangle?

QUESTION 13. Evaluate the following integrals:

a) $\int \ 7xe^{x^2+1} + \sqrt{x} + 4x \ dx$

a/2) $\int \ \frac{2x+1}{x^2+x+3} \ dx$

a/3) $\int \ \frac{x^3+x^2-6}{x^7} \ dx$

a/4) $\int \ (e^x + 1)\sqrt{e^x + x + 4} \ dx$

a/5) $\int \ \frac{1}{e^{-x}+8}$

a/6) $\int \frac{2xe^{x^2}}{(e^{x^2}+4)^4}\, dx$

QUESTION 14. Find y' and don't simplify:

a) $y = 3e^{(5x+4)} + ln(5x^2 + e^x + 7) + \frac{10}{x}$

b) $y = 4x(7x^3 + 2e^x)^3 + \sqrt{2x+7} + 3x^2$

c) $y = \sqrt[3]{7x+1}$

f) $y = ln[\frac{8x^2+7x-9}{(4x+e^{2x})^4}] + 10$

QUESTION 15. a) Find $lim_{x\to -2}\ \frac{e^{(3x+6)}-1}{3x^3-12x}$

b) Find $Lim_{x\to 5}\ \frac{\sqrt{3x+1}-4}{7x-35}$

c) $Lim_{x\to 0}\ \frac{3x^2}{e^x-x-1}$

QUESTION 16. Let $f(x) = 4e^{2x-6} + 3\sqrt{x-2} + ln(3x-8) - 1$

a) Find the equation of the tangent line to the curve of $f(x)$ when $x = 3$.
b) Use (a) to approximate y when $x = 2.7$.

QUESTION 17. a)Given $e^x + ln(2x+3y-8) + yx + 3y - 10 = 0$. Find the equation of the tangent line to the curve at $(0, 3)$.

b) Approximate the y-value when $x = 0.3$

QUESTION 18. a) Find the absolute maximum value of y and the absolute minimum value of y for $f(x) = -xe^{2x^2} + e^{2x^2} + 1$ defined on $[-1, 1]$.

b)For what values of x does $f(x)$ increase? and for what values of x does $f(x)$ decrease?

c) [if you like, it is only worth 2 points, i.e. if you do not feel like doing it, you only lose 2 points] Use only the concept of the first derivative and sketch a rough graph of $f(x)$

QUESTION 19. We want to construct a rectangle with maximum area such that two vertices on the line y = 4 and the other two vertices on the curve $y = 31 - x^2$. What should be the length and the width of such rectangle?

QUESTION 20. Evaluate the following integrals:

a) $\int 7e^{x+1} + \sqrt{x+1} + 4x\, dx$

a/2) $\int \frac{x+2}{x^2+4x+3}\, dx$

a/3) $\int \frac{x^5+x^6-12}{x^7}\, dx$

a/4) $\int (8e^x + 4)(4e^x + 2x + 4)^7\, dx$

QUESTION 21. a) A particle moves on the ellipse $x^2 + y^2/4 = 10$. The y is decreasing at rate 0.5 cm/sec. Find the rate of change of x at the point (3, 2).

b) a particle moves on the curve $xe^y + y^2 - 3xy + ln(3x - 8) + 7x = 24$ Find the rate of change of y at the point (3, 0) if the rate of change of x is 1cm/sec.

QUESTION 22. Find the length and the width of the largest rectangle that you can draw inside the ellipse $y^2 + x^2/4 = 25$ (two vertices on the x-axis and the other two vertices on the upper half of the ellipse)

QUESTION 23. Find the area of the region that is bounded by $f(x) = -x^2 + 5x - 6$ and the line $y = x - 3$ where $0 \leq x \leq 4$.

QUESTION 24. Let $x^2 + 2y^2 + xy - 4x + 8y + 4 = 0$. Find all the points on the curve where the tangent line at these points have slope equals 2.

Faculty information

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